Curriculum Framework

for Mathematics

Office of Curriculum, Instruction and Student Support
Instructional Services Branch

Department of Education
State of Hawaii

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FOREWORD

Broadly defined, curriculum is the total learning experience provided by a school to its students. It includes all of the content, goals and objectives, instructional materials, instructional strategies, student support and other services, and activities provided for students by the school.

Curriculum frameworks communicate common understandings about content and performance standards, instruction, and classroom assessment in a content area. The frameworks suggest ways that classroom instruction and assessment can be designed to best address the Hawaii Content and Performance Standards (HCPS) III. The curriculum frameworks also provide a means for schools to incorporate system-wide requirements into the school curriculum to ensure educational quality and equity for all students.

This framework is one of a series of Hawaii State Department of Education publications for teachers and other educators to use in implementing the HCPS III at the classroom level. Curriculum Frameworks for each of the nine HCPS III content areas provide a framework and philosophy for curriculum, instruction, and classroom assessment in those disciplines.

Patricia Hamamoto, Superintendent

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INTRODUCTION TO THE CURRICULUM FRAMEWORK SERIES

DESCRIPTION, PURPOSES, USES

Curriculum frameworks suggest the best thinking about the knowledge, skills, and processes that characterize a particular discipline; these frameworks provide a structure within which to organize curriculum and instruction in that content area. Curriculum frameworks represent the theoretical and philosophical bases, grounded in sound research, upon which the content standards, benchmarks, performance tasks, and rubrics were developed.

The curriculum framework series for the HCPS content areas include documents that provide the rationale or statements of the values, principles, research, and assumptions which help to guide decision making and the designing of curricular and instructional programs. Curriculum frameworks provide links between theory and practice as well as up-to-date and relevant information about pedagogy, learning, and resources within a content area.

Curriculum frameworks are intended for teachers and other educators and policy-makers involved in curriculum, instruction, and other educational decision-making. The frameworks are meant to provide a level of consistency, standardization, and equity in curriculum, instruction, and assessment across all classrooms across the state. The written format allows access to this information by all educators statewide.

Curriculum frameworks can be used by teachers as a roadmap to plan and design curricular and instructional units or activities at the school level and serve as aids in selecting appropriate classroom level materials for students as well as assessments that can be used for diagnosis, progress monitoring, and measuring outcomes. The frameworks can also serve as a common reference point in discussing and aligning curriculum schoolwide, or within a grade level or department.

THE SYSTEM OF STANDARDS

Fundamentally, standards provide *all* students with access to high expectations, challenging curricula, and effective teaching. Standards associate equity with excellence and ensure that students have the knowledge and skills necessary to participate in daily activities and in the workplace and to pursue their goals and aspirations.

The HCPS III describe educational targets in all nine content areas for *all* students in grades K-5. All students, therefore, are expected to be given the opportunity to meet all of the K-5 HCPS III standards. At the secondary level, however, the standards describe different things in different content areas. For the four CORE content areas (Language Arts, Mathematics, Science, and Social Studies) the standards describe expectations for all students, since all students are expected to take certain required courses in these areas. For the *extended core* (Health, Physical Education, Fine Arts, World Languages, and Career and Technical Education) they describe a continuum that should be expected by students who choose courses in these areas as electives. It should be emphasized that *all* courses, required or elective, are standards-based and are part of the *Hawaii Standards System*.

THE HAWAII STANDARDS SYSTEM

The Hawaii Standards System is more than the HCPS III alone. The Hawaii Standards System supports standards-based education through curriculum, instruction and assessment components. The system also provides student instructional support components such as Special Education and English for Second Language Learners. It also includes student and family support components such as Pihana Na Mamo and Parent Community Network Coordinators. The Hawaii Standards System supports school level implementation of standards-based education by

- Identifying the targets for student learning such as the Vision of the Public School Graduate, General Learner Outcomes, the HCPS III, and other course standards;
- Providing curricular and behavioral support for students through direct services to students and their families; and
- Developing, acquiring, and assuring access to support for implementation of standards-based education for teachers, school leaders, and other academic staff.

The HCPS III contain

- Essential content and skills in *nine* content areas: Career and Technical Education, Fine Arts, Health, Language Arts, Math, Physical Education, Science, Social Studies, and World Languages;
- Standards that describe the educational expectations for ALL students in grades K-5

- Essential standards for all required courses in the *four core* areas: Language Arts, Math, Science, and Social Studies; and
- Essential standards that can be met through elective courses chosen by secondary students to fulfill graduation requirements in the *five extended core* areas: Career and Technical Education, Fine Arts, Health, Physical Education, and World Languages.

Included in the Hawaii Standards System are standards for courses not found in this HCPS III documents. These standards may be found in HCPS II and will be identified in a future version of the *Approved Course and Code Numbers* (ACCN) course descriptions. Because *all* courses are standards-based, these specialized courses utilize

- Industry or national standards that describe essential content and skills for elective courses in areas such as Career and Technical Education and Fine Arts; and
- Content area-specific standards found in HCPS II.

THE RELATIONSHIP BETWEEN THE STANDARDS AND THE GENERAL LEARNER OUTCOMES

Content Standards define the academic content knowledge and skills that all students should know and be able to do. They are general statements of expectations for all students K-12.

Equally important to learning academic content is developing the knowledge, skills and attitudes that all students need in order to lead full and productive lives. The six General Learner Outcomes (GLOs) serve as the essential, overarching goals in the system of standards. These Outcomes are

- GLO 1: Self-directed learner: The ability to be responsible for one's own learning
- GLO 2: Community Contributor: The understanding that it is essential for human beings to work together
- GLO 3: Complex Thinker: The ability to be involved in complex thinking and problem solving
- GLO 4: Quality Producer: The ability to recognize and produce quality performance and quality products
- GLO 5: Effective Communicator: The ability to communicate effectively
- GLO 6: Effective and Ethical User of Technology: The ability to use a variety of technology effectively and ethically

These Outcomes must be an integral part of teaching and learning and the heart of every Hawaii classroom. Teachers of all subjects in all grades must contribute to the development of the GLOs while promoting the learning of subject matter as well.

The real test of the standards is their ability to improve student learning. Raising expectations is but the first step; it is what is done with the standards—how they are realized

in all classrooms for all students—that will determine whether the Department can fulfill its vision of graduating students who

- realize their individual goals and aspirations;
- possess the attitudes, knowledge and skills necessary to contribute positively and compete in a global society;
- exercise the rights and responsibilities of citizenship; and
- pursue post-secondary education and/or careers without need for remediation.

THE HCPS III IMPLEMENTATION MODEL

The Hawaii Content and Performance Standards (HCPS) III Implementation Process Model is a framework that has been adapted from West Ed's Learning from Assessment model. It consists of a series of six steps.

- The first step in the process asks a teacher to identify relevant benchmarks. The teacher decides which benchmarks will be the central focus of a lesson or unit.
- In the second step, the teacher determines what evidence will show that the students have met the benchmarks.
- In the third step of the process, the teacher plans the strategies and experiences which will build understanding and help all students meet proficiency.
- The fourth and fifth steps require the collection of evidence of student learning. The teacher determines what this evidence indicates about the student's progress and decides what further instruction or support is needed.
- Lastly, the teacher evaluates the work and communicates the findings.

While the model numbers the steps in the process, it is important to remember that these steps are not always followed in a lock-step fashion. For example, a teacher may work through steps one to five, and as she collects the evidence of student learning (step five), she will likely gain insight that will inform step three (determine learning experiences). In her review of the work, she may notice that many students are not meeting a certain aspect of a particular benchmark. For example, the students may be able to correctly compare fractions, but may be unable to explain why they placed the fractions in a particular order. This evidence will inform step three and the teacher will likely design additional learning experiences designed to help students place fractions in a particular order.

HCPS III IMPLEMENTATION PROCESS MODEL

HCPS III IMPLEMENTATION STEPS
❶ Identify relevant benchmarks.
❷ Determine acceptable evidence and criteria.
❸ Determine ***learning experiences*** that will enable students to learn what they need to know and to do.
❹ Teach and collect evidence of student learning.
❺ Assess student work to inform instruction or use data to provide feedback.
❻ Evaluate student work and make judgment on learning results and communicate findings.

Reteach or repeat the process with the next set of benchmarks.

The table on the next page shows the six-step HCPS III Implementation Process Model. It also shows the state and school support for student success that relates to each step in this model.

HCPS III IMPLEMENTATION PROCESS MODEL

Implementation Steps	State Support for Student Success	School Support for Student Success
❶ Identify relevant benchmarks. *Which benchmarks will be the central focus of the lesson/unit?*	**Benchmark Map** (http://standardstoolkit.k12.hi.us) ~ developed by State with input from field ~ includes sets of benchmarks clustered around Big Ideas or Major Understandings; clusters mapped out by quarters ~ serves as the focal point for other state-developed supporting documents and future standardized course assessments and HSA	**Curriculum Map** [Lotus Notes curriculum mapping program available at no cost (check with your principal)] ~ developed by teachers/schools to create a cohesive and articulated curriculum ~ aligned to Benchmark Map
❷ Determine acceptable evidence and criteria. *What evidence will show that the student has met the standards?*	**Instructional Map** ~ will be developed by OCISS with input from field ~ aligned to Benchmark Map ~ includes sample assessment tasks and rubrics	**Curriculum Map** (continued) ~ includes assessment tasks (may include teacher-developed tasks, or tasks from the Instructional Map, textbook, journals, publications, websites, or other resources)
❸ Determine ***learning experiences*** that will enable students to learn what they need to know and to do. *What strategies/experiences will build understanding and help all students meet proficiency?*	**Instructional Map** (continued) ~ will include sample instructional strategies to provide opportunities for ALL students to reach proficiency **Instructional Materials Review** ~ development of Recommended Textbook List that includes resources that support standards-based instruction and assessment	**Unit/Lesson Plans** ~ developed by teachers ~ aligned to Curriculum Map ~ learning experiences may come from a variety of resources: Instructional Map, textbooks, journals, publications, websites, or other resources ~ includes plans for formative assessment
❹ Teach and collect evidence of student learning. ❺ Assess student work to inform instruction or use data to provide feedback. *What does the evidence indicate about the student's progress?* *What further instruction or support is needed?*	**Instructional Map** (continued) ~ will eventually include student work (exemplars) for the tasks that are provided	**Formative Assessments (from Step #3)** ~ used to guide instruction and inform students of their progress **Summative Assessments (from Step #2)** ~ used to assess student's level of proficiency after the student has had a chance to learn, develop, and improve
❻ Evaluate student work and make judgment on learning results and communicate findings. *What do recent assessments indicate about the student's level of proficiency?*	**Standardized Course Assessments** ~ coming soon for high school courses	**Standards-Based Grading and Reporting** ~ used to report progress/proficiency of benchmarks that were identified in Step #1
Reteach or repeat the process with the next set of benchmarks.		

THE STANDARDS-BASED CLASSROOM

The standards-based classroom does not have one particular form. Rather, it can take on many forms. Characteristics to look for include the following.

What are students doing?

- Working in collaborative groups, talking and sharing ideas about the subject matter and solving problems or conducting investigations together
- Listening actively to each person's ideas and being critical friends when someone needs help understanding a difficult concept
- Demonstrating persistence in performing complex tasks and learning challenging concepts
- Communicating thoughts, ideas, findings, and solutions to others
- Using and knowing when to use various resources (such as printed materials, tools, and technology) to learn about the subject matter
- Reflecting on their progress toward learning goals

What are teachers doing?

- Asking good questions to get students to think more deeply about a posed problem or task
- Constantly assessing where students are with respect to the focus of the lesson and adjusting the lesson based on feedback about student understanding
- Creating a climate for risk-taking and encouraging subject-matter dialogue where students exchange a variety of ideas and feel confident about asking questions
- Providing opportunities for students to learn at their own pace using strategies for differentiation
- Using text materials, tools, technology, multimedia, guest speakers, and/or field experiences to enhance learning
- Making every effort to show links between and among disciplines and how the subject matter is connected and relevant to other areas and real contexts

REFERENCE

Jamentz, K. (1998). *Standards: From document to dialogue.* San Francisco, CA: WestEd

PREFACE

In 2005, the Office of Curriculum, Instruction and Student Support, Instructional Services Branch published and disseminated the HCPS III mathematics standards. The mathematics content in these standards represent what all students need to know and be able to do to have choices for post-high school paths and to succeed in a rapidly-changing technological and global society. The need to understand and use mathematics as a tool for solving problems, for making informed decisions, and for creating new technologies is critical in today's environment. The value placed on strategies for ensuring that the mathematics education of our students is challenging and rigorous is demonstrated by numerous international and national initiatives that address strengthening the implementation of national and local standards to improve student learning and achievement in mathematics. The Department believes that by strengthening the implementation of Hawaii's standards, students will be better prepared to meet the challenges of today's societal needs. It is hoped that this framework will provide the teaching community with the content and process guidelines for enabling students to make the vision of a Hawaii high school graduate a reality.

The *Curriculum Framework for Mathematics* will provide the rationale for the content delineated in the HCPS III mathematics standards, a description of the entire standards system and the processes for implementing that system, and samples of the major elements of the system and commentary on their use.

The intent of this Framework is to guide teachers and administrators in acquiring the tools and materials for implementing the standards, providing the instructional strategies for helping students attain and exceed the standards, conducting the assessment necessary to guide instruction to better meet needs of students, reporting student progress to communicate to all involved on the status of students meeting standards, and involving multiple role groups in helping to educate the children in their communities.

1. GENERAL DESCRIPTION OF THE MATHEMATICS PROGRAM

DEFINITION OF THE MATHEMATICS PROGRAM

> *Mathematics is essentially a process of thinking that involves building and applying abstract, logically connected networks of ideas. These ideas often arise from the need to solve problems in science, technology, and everyday life—problems ranging from how to model certain aspects of a complex scientific problem to how to balance a checkbook.*
>
> *—Science for All Americans*

Mathematics is a branch of science—while science is a way of interpreting and describing the environment in which we live, mathematics is the study of patterns and relationships that exist around us. In Hawaii, beachgoers know the recurring pattern of the appearance of jellyfish on the south shores three days after the appearance of a full moon. Forensics specialists can determine the height of a man based on the length of his femur. A car owner can evaluate the fuel efficiency of her car by keeping track of the distance traveled prior to each fill-up of the gas tank.

The mathematics involved in understanding the patterns and relationships around us require students to become critical thinkers and problem solvers. The problems that adults face everyday do not come with a textbook that tells us what mathematical concepts are needed to solve that problem; we need to first identify the problem, determine what mathematics are needed to solve the problem, apply mathematics to solve the problem, evaluate the results, and then act on the results.

Mathematics education serves to arm students with the knowledge and skills necessary to become efficient and effective problem solvers. The mathematical knowledge and skills developed from pre-kindergarten through grade 12 are identified in the Hawaii Content and Performance Standards III Mathematics Standards and are organized around five major strands: Number and Operations; Measurement; Geometry and Spatial Sense; Patterns, Functions, and Algebra; and Data Analysis, Statistics and Probability.

The mathematics curriculum should include more than just the mathematics standards. To achieve the vision of a public school graduate, lessons must be designed to provide opportunities for students to practice the General Learner Outcomes: 1) self-directed learner, 2) community contributor, 3) complex thinker, 4) quality producer, 5) effective communicator, and 6) effective and ethical user of technology. In addition, mathematics has its own set of process standards that are related to the General Learner Outcomes and also serve to enhance instruction: 1) problem solving, 2) communication, 3) representation, 4) reasoning and proof, and 5) making connections. When integrated with teaching the

mathematics standards, the General Learner Outcomes and the process standards provide a positive learning experience for **all** students.

RATIONALE FOR THE MATHEMATICS PROGRAM

The role of mathematics in everyday life and in the work place has gone through major changes within the last decade. The need to understand and be able to use mathematics has never been greater. The challenges posed by a highly technical world—in which technologies change so rapidly yet demand a citizenry able to maintain the pace of change while creating new technologies—require education that teaches students how to use mathematics as a tool to solve any problem as well as the ability to think mathematically to create new ideas.

Understanding that students exhibit different talents, abilities, achievements, needs and interests in mathematics, we must be adaptive to the differences among students. Students with a deep interest in mathematical and scientific careers require opportunities to engage their talents and interests, while students with special educational needs require appropriate support to achieve proficiency of the mathematics standards so that they too can aspire to reach the goals they set for themselves. In order to provide all students with access to the highest quality mathematics education, the mathematics program must use research-based and data-driven approaches to improve, maintain, and support a system of standards-based curriculum, instruction, and assessment.

BELIEFS AND ASSUMPTIONS

Teaching mathematics is complex, contextual, and requires a commitment to the belief that *all* students can learn challenging mathematics. The attitude that "It's OK if you can't do math, because I couldn't do math either, and I'm successful" or "Not everyone can do math" needs to be dispelled. It is the teacher's responsibility to determine what needs to be done in classroom instruction to ensure that all students have the opportunity to learn and demonstrate that they have met the expectations set by the standards.

In addition to the fundamental belief that all students can learn, and the assumption that all students want to learn, the Department believes in the following principles set forth by the National Council of Teachers of Mathematics:

- ***Equity***. Excellence in mathematics education requires equity—high expectations and strong support for all students.
- ***Curriculum***. A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across grades.
- ***Teaching***. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- ***Learning***. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

- ***Assessment***. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- ***Technology***. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

The expectations of Hawaii's content standards require teachers to have strong mathematical content background and experiences with the concepts, procedures and skills addressed by the standards. Possessing a strong content background enables teachers to design mathematical tasks that are rich and engaging; develop important and relevant mathematical concepts and skills; find connections within and outside of mathematics that are coherent with Hawaii's standards; support problem identification and formulation, problem solving, and mathematical reasoning; promote communication about mathematical ideas; and stimulate positive attitudes and feelings toward mathematics. Such teachers are especially needed to provide struggling learners with the kind of learning environment that fosters at-grade-level learning rather than relegating those students to remedial mathematics classes.

RESEARCH

Decades of research in the cognitive and developmental sciences have provided the foundation for what we know today about the way students learn. The National Research Council set forth three fundamental principles of learning:

1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.
2. To develop competence in an area of inquiry, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.
3. A "metacognitive" approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

These three principles mirror the expectations of the General Learner Outcomes and validate the importance of integrating the teaching of the mathematics standards with the development of the General Learner Outcomes. At first, the General Learner Outcomes may seem like something "extra" that teachers need to teach. But by taking a metacognitive approach to teaching (GLO 1: Self-Directed Learner; GLO 4: Quality Producer), helping students to connect new knowledge with prior experiences (GLO 3: Complex Thinker), and providing the learning experiences needed to develop and maintain new understandings (GLO 2: Community Contributor; GLO 5: Effective Communicator; and GLO 6: Effective and Ethical User of Technology), greater proficiency of the mathematics standards will result.

LEGAL AUTHORITY

The legal authority for the Mathematics program is described in the Department's policies and regulations, Curriculum and Instruction Series 2000 handbook. Relevant Board of Education policies and Department of Education regulations based on those policies are listed here.

ACADEMIC PROGRAM
(HAWAII STATE BOARD OF EDUCATION POLICY 2100)

The Board of Education recognizes that one of the key components to student achievement and success is a quality, standards-based academic program. Therefore, the Department of Education shall provide an academic program to equip each student with the knowledge, skills, attitudes, and values needed to attain the Hawaii Content and Performance Standards and to give responsible direction to one's own life. The Department of Education shall provide standards-based learning experiences to develop and nurture a variety of intelligences.

Effective learning shall be facilitated through the maximum and active participation of each student in the learning process, insuring that personal meaning is derived from curriculum content, appropriate and relevant teaching and learning strategies, and self-assessment as well as standards-based assessment, grading and reporting procedures. The learning experiences shall be included in concepts commonly taught in, but not limited to, language arts, mathematics, science, social studies, health, physical education, fine arts, world languages, and career and life skills, or a combination of the above subject areas.

Each school shall offer a comprehensive program of academic education to meet the needs, interests, and abilities of all students.

Adopted: 10/70
Amended: 08/86; 03/88; 01/99; 01/05/06

ACADEMIC PROGRAM
(HAWAII STATE BOARD OF EDUCATION REGULATION 2100.1)

1. It is the right of every student to have access to a learning program which will permit optimum development as an educated person.
2. The academic program shall include a desirable mix of appropriate and comprehensive learning activities in the areas of (a) communications, (b) humanities, and (c) environmental studies.
3. The basic program, to be offered at each school, shall consist of the knowledge, skills and processes, and attitudinal development to be required of each student as the foundation for attainment of higher academic learning.
4. The minimum elective program enhances the basic program and consists of desirable courses in the major subject areas which may be scheduled in accordance with student interest, staffing and related considerations.
5. The specialized elective program, which shall be planned to meet the unique needs and interests of students and school committees, shall reflect current and emerging concerns of the community, the nation, and the world.

Adopted: 10/70
Amended: 8/86, 3/88

K-12 LITERACY
(HAWAII STATE BOARD OF EDUCATION POLICY 2010)

The development of student literacy in all content areas and in all grade levels is an educational and cultural imperative. Literacy shall be attained through an appropriate framework of curriculum and instruction. Literacy is the ability in any content or context to read, write, and communicate. Literacy shall include mathematical and scientific literacy. Other skills that enhance literacy include relating, expressing, speaking, understanding, listening, critical thinking, analyzing, and problem-solving.

The language arts standards in the Hawaii Content and Performance Standards specify what all students should know and be able to do to become literate. To attain this goal, all schools shall provide a balanced and comprehensive reading and writing program that includes the direct teaching of: (1) comprehension of content and language in both oral and written forms; (2) organized and explicit skills instruction, that includes phonemic awareness, phonic analysis, and decoding skills, especially in the early grades; and (3) fluency and vocabulary development that includes an understanding of how words work. The reading and writing program shall also provide: (4) ongoing diagnosis and assessment that ensures accountability for results; (5) effective writing practices to be integrated into the reading and writing program; and (6) timely intervention services to assist students who are at risk of failing attainment of literacy.

An effective early reading and writing program shall be implemented to assure that every child will become a proficient reader and writer, as defined by the Department of Education, by the end of third grade.

In the instructional program for grades 4-12, all content areas shall further support the development of literacy skills such that students can access and communicate subject area content and concepts using a wide variety of print and non-print materials.

Students identified by the Department of Education as not proficient will receive appropriate assistance and support.

Adopted: 10/94 (Curriculum and Instructional Policy)
Amended: 4/98; 6/02; 10/19/06.

CURRICULUM AND INSTRUCTION IN THE EDUCATION PROCESS (HAWAII STATE BOARD OF EDUCATION REGULATION 2010.1)

The roles of the curricular and instructional programs for the public schools of Hawaii shall be both broad and inclusive, bringing focus to experiences which will equip students for a lifetime of effective living and learning, permitting them to meet successfully today's problems and opportunities as well as on those in the yet-unknown future.

Curriculum and instruction shall provide experiences which will enable students to learn to think and act intelligently in achieving maximum self-fulfillment and in attaining the knowledge, skills, abilities, attitudes, and appreciations essential for preserving and contributing to the strength of the community, state, nation, and world.

Effective learning shall be predicated on maximum participation of each student in the learning process, insuring that personal meaning is derived from curriculum content, instructional modes, and evaluative procedures.

Provisions shall be made for incorporating many diverse experiences throughout the school years to assist learners in realizing to the fullest their unique potentialities, as well as to make certain that appropriate attention is directed toward the problems and progress of society. The emphasis and degree of sophistication of these experiences shall be appropriate to the needs and characteristics of the learners.

School experiences which contribute to self-fulfillment and productive life shall include the following:

1. Development of basic skills for learning and communication, including, speaking, reading, writing, listening, computing, and thinking.
2. Development of positive self-concept, including understanding and accepting self and understanding and relating effectively with others.
3. Development of decision-making and problem-solving skills.
4. Development of independence in learning, including demonstrating initiative and responsibility for continuous learning.
5. Development of physical, social and emotional health, including demonstrating good health, fitness and safety practices.
6. Recognition and pursuit of career development as an integral part of growth and development.

7. Development of a continually growing philosophy based on beliefs and values and including responsibility to self and others.
8. Development of creative potential and aesthetic sensitivity.

Adopted: 10/70
Amended: 3/88, 10/94

HAWAII CONTENT AND PERFORMANCE STANDARDS
(HAWAII STATE BOARD OF EDUCATION POLICY 2015)

To ensure high academic expectations, challenging curriculum, and appropriate assessment and instruction for all students, the Department of Education shall implement the Hawaii Content and Performance Standards as approved by the Board of Education. The standards shall specify what students must know and be able to do.

Schools shall articulate and align their curricular, assessment and instructional program—by grade level, subject area, courses, and/or other appropriate units—with the Hawaii Content and Performance Standards and evaluate the effectiveness of their efforts to help all students attain the standards. The school's articulated curricular, assessment and instructional program shall be shared with parents and students with the intent of involving parents/guardians as partners in the education of their children.

The Superintendent shall develop and implement a plan to create a standards-based and performance-oriented education system that will ensure that all students attain the standards.

Approved: 10/95
Amended: 11/01; 06/23/05

HAWAII CONTENT AND PERFORMANCE STANDARDS
(HAWAII STATE BOARD OF EDUCATION REGULATION 2015.1)

1. The Hawaii Content and Performance Standards shall be implemented as approved by the Board of Education and distributed to the schools.
2. Each school shall describe its implementation of the standards in its Standards Implementation Design (SID).
3. The Department of Education shall develop and implement a continuum of professional development activities that enable teachers to implement the standards.
4. The Department of Education shall develop an assessment and accountability system that measures and reports on student attainment of the standards and holds everyone accountable for that performance.
5. The Department of Education and the Board of Education shall coordinate the review and revision of the standards every five years.

DOE: 11/01

RESPONSIBILITY FOR CURRICULUM DEVELOPMENT AND IMPLEMENTATION
(HAWAII STATE BOARD OF EDUCATION POLICY 2030)

The Department of Education shall provide guidance to schools in developing and implementing curriculum and instruction for the public school system.

The responsibility for developing curriculum shall be shared by the Superintendent and the schools. The responsibility for developing and delivering the instructional program shall rest primarily with the schools. The Superintendent shall provide the general direction in curriculum and instruction by providing guidance in the use of effective teaching, learning, and assessment strategies appropriate to the Hawaii Content and Performance Standards.

Former Code No. 6123.2
Former Policy Approved: 07/60
Amended: 10/70, 03/88; 03/99

CURRICULUM DELIVERY
(HAWAII STATE BOARD OF EDUCATION POLICY 2101)

The Board of Education recognizes that a strong, challenging curriculum is key to student success and achievement. Therefore, all elementary (grades K-5) and secondary schools (middle/intermediate and high) shall design a program of studies—or curriculum—that enables all students to attain, to the highest degree possible, the Hawaii Content and Performance Standards (HCPS). The curriculum shall include:

- Units of study or lessons, delineating content or topics to be taught;
- Relevant instructional activities and materials to be used, aligned with the HCPS;
- Specific learner outcomes or expectations that result in student attainment of grade level benchmarks;
- A timeframe in which outcomes are expected to be achieved; and
- Assessment tools and methods, including collection and analysis of student work, to measure student attainment of outcomes and benchmarks.

With continued emphasis on improving student achievement, the articulation and coordination of curriculum and curricular services between and among grade levels and subject areas shall be addressed at every school. Articulation of services between schools within a complex shall also be addressed.

The curriculum or program of studies shall include academic courses, subjects, and/or units as well as planned, systematic co-curricular activities and student academic support services, such as assessment, counseling, and guidance to facilitate student attainment of standards. The Department of Education shall adopt regulations to assist schools in the implementation of this policy.

Approved: 11/03/05

INSTRUCTIONAL MATERIALS
(HAWAII STATE BOARD OF EDUCATION POLICY 2240)

The Board of Education understands that implementation of standards-based education requires instructional materials that are aligned with the Hawaii Content and Performance Standards (HCPS). Therefore, printed materials, media and technology which overtly address the HCPS benchmarks shall be selected for classroom use.

The Office of Curriculum, Instruction and Student Support shall provide a list of recommended textbooks and other instructional materials for select curricular areas. It shall also provide general and content-specific evaluation criteria for schools to use when evaluating instructional materials.

Schools that select texts and instructional materials not on the list of recommended texts and instructional materials shall demonstrate that these materials will better support their students' learning needs. Evidence shall include statewide assessment results and other data documenting student achievement.

Schools shall also develop and implement a multi-year textbook acquisition/replacement plan that is based on instructional needs. This shall be a key component of a schools' academic and financial plan. Schools shall inform parents and make available to their school communities, the textbook acquisition/replacement plan, its adequacy in meeting students' needs for textbooks in a given year, and the textbook series, by subjects, used in classrooms.

Former Code Nos. 6134 Textbooks and Reference Materials
6134.1 Approval of Reference Materials Offered by Special Interest Group
Former Policy 6134.1 Approved 01/55; Reviewed 07/60; Revised and included above 4/70
Approved: 10/70
Amended: 03/88; 05/95; 03/97; 09/98; 01/05/06

PROGRAM GOALS

VISION

Students who value mathematics and engage actively in learning and using it.

MISSION

To provide leadership and professional improvement to support teachers in ensuring the highest quality mathematics education for all students.

GOALS

Our goal for mathematics education: **Mathematical proficiency for *all* students**.

Mathematical proficiency has five interwoven and interdependent components:

1. *Conceptual understanding*: Comprehending mathematical concepts, operations, and relations—knowing what mathematical symbols, diagrams, and procedures mean.

2. *Procedural fluency*: Carrying out mathematical procedures (such as adding, subtracting, multiplying, and dividing numbers) flexibly, accurately, efficiently, and appropriately.

3. *Strategic competence*: Being able to formulate problems mathematically and to devise strategies for representing and solving them using concepts and procedures appropriately.

4. *Adaptive reasoning*: Having the capacity for logical thought, reflection, explanation, and justification; using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.

5. *Productive disposition*: Seeing mathematics as sensible, useful, doable, and worthwhile, coupled with a belief in diligence and one's own efficacy.

For all students to achieve success in learning mathematics, the five components must be developed concurrently, and not one at a time. For example, to develop procedural fluency without the other four components means that students are just performing procedures without understanding how or why they work and in what context they would apply this procedure; while these students may be able to perform the procedures for the moment, they lack the understanding to apply or adapt this skill to new situations.

At the same time the five components of mathematical proficiency are addressed, students will be simultaneously engaging the National Council of Teachers of Mathematics (NCTM) process standards, as well as the General Learner Outcomes. (See table on the following page.) The process standards and the General Learner Outcomes were introduced in HCPS II, and continue to be emphasized in HCPS III, along with the components of mathematical proficiency. All three are linked together, and serve as the heart of standards-based mathematics instruction throughout all grade levels and mathematics courses.

Components of Mathematical Proficiency	The NCTM Process Standards	General Learner Outcomes
Conceptual understanding	• Communication • Representation • Reasoning and Proof • Making Connections • Problem Solving	1. Self-Directed Learner 2. Community Contributor 3. Complex Thinker 4. Quality Producer 5. Effective Communicator 6. Effective and Ethical User of Technology
Procedural fluency		
Strategic competence		
Adaptive reasoning		
Productive disposition		

2. THE MATHEMATICS STANDARDS

THE NEED FOR STANDARDS

Standards serve as a catalyst or guide for focused and sustained improvement in mathematics education. The mathematics content standards are comprehensive and broad goals that identify what students should know about mathematics and be able to do using mathematics in order to make sense of the world around them. Standards ensure that ***all*** students are provided the opportunities and support necessary to learn significant mathematics with depth and understanding and to become mathematically literate.

The mathematics standards are developed

1. *To promote equity and excellence for all.* Standards for all students are necessary to promote high expectations and to ensure equitable educational opportunities—qualified teachers, access to resources, and a fair opportunity to learn toward high expectations. The National Council of Teachers of Mathematics (NCTM) in *Principles and Standards for School Mathematics* (2000, 4) makes clear that standards

   > ". ..play a leading role in guiding the improvement of mathematics education. As an organization representing teachers of mathematics, NCTM shares with students, school leaders, and parents and other caregivers the responsibility to ensure that all students receive a high-quality mathematics education. All interested parties must work together to create mathematics classrooms where students of varied backgrounds and abilities work with expert teachers, learning important mathematical ideas with understanding, in environments that are equitable, challenging, supportive, and technologically equipped for the twenty-first century."

2. *To ensure that students are mathematically literate to meet the demands of society now and in the future.* The future will require mathematically-literate citizens who can function in a highly technological and communication-oriented society. Mathematical competencies once achieved by a few will be needed by all to live and contribute successfully to society. These competencies go beyond the ability to do arithmetic and solve simple problems. These competencies will include the ability to make decisions based on data, to use technology to represent and solve complex problems, and to communicate verbally and symbolically the processes and information required for various situations.

THE SETTING OF THE STANDARDS

The HCPS III mathematics standards and benchmarks were developed with the HCPS II as the starting point. The Mathematics Section of the DOE worked with the Mid-continent Research for Education and Learning (McREL) through the development process. McREL is one of ten regional educational laboratories that make up the Regional Educational Laboratory System, which serves education agencies and schools across the nation. McREL is known for its extensive work with standards and is at the forefront of standards-based education. The following specifications were followed in the development of HCPS III standards.

- Essential standards, benchmarks and performance indicators were used as the foundation for the HCPS III standards.
- As the new standards statements were developed, they were also compared against national standards and other highly regarded state's standards.
- Analysis of the standards led to the elimination of overlaps and/or redundancies within and between content areas.
- Consistent grain size (benchmarks that were of approximately the same instructional size)
- Standards, benchmarks, sample performance assessments, and rubrics were written in plain language, understandable to primary audience (teachers) and secondary audiences (students and parents).
- Benchmarks were written as describing "proficient." Attention was paid to the taxonomic level of the benchmarks so that they would appropriately scaffold and challenge students.
- Implementable—The benchmarks were written with consideration of the delivery of instruction (integrated elementary curriculum, required and elective courses at the secondary level)
- Benchmarks and sample performance assessments were written to be measurable through the examination of student work from which valid inferences about student learning could be made

The refinement process occurred in three phases and involved an array of professionals. In the first phase, classroom teachers and resource teachers reviewed the benchmarks developed by McREL to ensure that the benchmarks (a) incorporated relevant and appropriate concepts and skills, (b) were measurable, and (c) were consistent with the standards identified in the National Council of Teachers of Mathematics document, *Principles and Standards of School Mathematics* (2000). In the second phase of the refinement process, university professors were invited to attend an HCPS III forum, and each of the fifteen Complex Area Superintendents sent a team of educators. The focus of the forum was to provide a mixed group of educators an opportunity to provide additional feedback on the benchmarks and the sample performance assessments. Then in the third phase, other DOE personnel (program managers, other content specialists) provided feedback regarding readability to the general educator. Input from all phases of the refinement process, and recommendations from

McREL led to the final version of the HCPS III mathematics standards, benchmarks, sample performance assessments, and rubrics.

THE ORGANIZATION OF THE STANDARDS

The expectations of what students should know and be able to do are stated as grade-level or course *benchmarks* and are organized in fourteen *content standards* under five content *strands* (see Appendix A: Standards at a Glance). While the benchmarks are specific to grade levels and courses, they serve as the stepping stones to meeting the K-12 expectations of the content standards. The nouns within a benchmark often indicate the content knowledge that is being developed. The verbs, which are equally important, tell what the students are expected to do with the content. The mathematics process standards of Communication, Connections, Problem Solving, Representation, and Reasoning and Proof have been thoughtfully embedded in the benchmarks and will become more obvious and overt in subsequent instructional documents. The General Learner Outcomes are linked with these process standards and should be emphasized in instruction.

STANDARDS-AT-A-GLANCE

The key features of the HCPS III mathematics standards are captured in the chart on the next page.

STRAND	STANDARDS	• TOPICS
NUMBER AND OPERATIONS	**Standard 1: NUMBER SENSE** Understand numbers, ways of representing numbers, relationships among numbers, and number systems	• Numbers and Number Systems • Vectors
	Standard 2: OPERATION SENSE Understand the meaning of operations and how they relate to each other	• Operations • Operation Properties
	Standard 3: COMPUTATION STRATEGIES Use computational tools and strategies fluently and, when appropriate, use estimation	• Computational Fluency • Estimation • Vectors
MEASURE-MENT	**Standard 4: FLUENCY WITH MEASUREMENT** Understand attributes, units, and systems of units in measurement; and develop and use techniques, tools, and formulas for measuring	• Measurement Attributes and Units • Measurement Tools and Techniques • Measurement Formulas
GEOMETRY AND SPATIAL SENSE	**Standard 5: PROPERTIES AND RELATIONSHIPS** Analyze properties of objects and relationships among the properties	• Geometric Shapes and Their Properties and Relationships
	Standard 6: TRANSFORMATIONS AND SYMMETRY Use transformations and symmetry to analyze mathematical situations	• Transformation • Symmetry
	Standard 7: VISUAL AND SPATIAL SENSE Use visualization and spatial reasoning to solve problems both within and outside of mathematics	• Visualization and Spatial Reasoning • Geometric Modeling
	Standard 8: REPRESENTATIONAL SYSTEMS Select and use different representational systems, including coordinate geometry	• Coordinate Geometry
PATTERNS, FUNCTIONS, AND ALGEBRA	**Standard 9: PATTERNS AND FUNCTIONAL RELATIONSHIPS** Understand various types of patterns and functional relationships	• Patterns • Function
	Standard 10: SYMBOLIC REPRESENTATION Use symbolic forms to represent, model, and analyze mathematical situations	• Numeric and Algebraic Representations • Rates of Change
DATA ANALYSIS, STATISTICS, AND PROBABILITY	**Standard 11: FLUENCY WITH DATA** Pose questions and collect, organize, and represent data to answer those questions	• Data Collection and Representations
	Standard 12: STATISTICS Interpret data using methods of exploratory data analysis	• Data Interpretation
	Standard 13: DATA ANALYSIS Develop and evaluate inferences, predictions, and arguments that are based on data	• Predictions and Inferences
	Standard 14: PROBABILITY Understand and apply basic notions of chance and probability	• Probability
The process standards of Communication, Connections, Problem Solving, Representation, and Reasoning and Proof are incorporated throughout the above mathematics standards.		

DESCRIPTION OF THE STRANDS

The **Number and Operations** strand describes deep and fundamental understanding of and proficiency with counting, numbers, and arithmetic, as well as an understanding of number systems and their structures. The central focus is the development of number sense—the ability to decompose numbers fluently, use referents like 100 or 1/2, use relationships among arithmetic operations to solve problems, understand the base-ten system, estimate, and make sense of numbers and recognize the relative and absolute magnitude of numbers. Computational fluency is essential. Computational fluency should be developed at the same time with mathematical understanding.

Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil. At more sophisticated levels, measurement involves assigning a number to a characteristic of a situation. Understanding what a measurable attribute is and becoming familiar with the units and processes that are used in measuring attributes is a major emphasis in this strand.

The **Geometry and Spatial Sense** strand offers students ways to interpret and reflect on the physical environment and can serve as tools for the study of other topics in mathematics and science. Geometric shapes and structures and how to analyze their characteristics and relationships, visual and coordinate representations, and explorations with motion (transformation) are central to the standards in this strand.

The **Patterns, Functions, and Algebra** strand emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change. Much of the symbolic and structural emphasis in algebra can build on students' extensive experiences with number.

The **Data Analysis, Statistics, and Probability** strand recommends that students formulate questions that can be answered using data and addresses what is involved in gathering and using data wisely. Students should learn how to collect data, organize their own or others' data, display data in graphs and charts, analyze data, and make inferences and conclusions from data. Concepts and applications of probability are also part of this strand.

The process standards of Problem Solving, Reasoning and Proof, Communication, Connections, and Representation are incorporated throughout the above content strands.

Problem Solving is an integral part of all mathematics learning. Problem solving means engaging in a task for which the solution is not known in advance. Good problem solvers analyze situations carefully in mathematical terms and pose problems based on situations they see.

Reasoning and Proof must be a consistent part of students' mathematical experiences. Through the use of reasoning, students learn that mathematics makes sense. Reasoning mathematically is a habit of mind and must be developed through consistent use in many

contexts. Proofs and logically rigorous deductions of conclusions from hypotheses provide the opportunity for students to appreciate the value of such arguments.

Communication allows students to reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged to express themselves clearly and coherently, both orally and in written form.

Emphasizing mathematical **Connections** helps students recognize how ideas in different areas of mathematics are related. Students should be given opportunities to experience mathematics in context—to connect mathematical concepts to their daily lives as well as to situations in other disciplines such as science, social studies, medicine, and commerce.

Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others. They allow students to recognize connections among related concepts and apply mathematics to real-world problems. Representations such as diagrams, graphical displays, and symbolic expressions are tools for learning and doing mathematics.

3. CURRICULUM, ASSESSMENT AND INSTRUCTION

In a standards-driven system, teaching becomes more complex. Traditionally, teaching, especially in mathematics classrooms, followed a formula—correct homework in the first five to ten minutes, teach a new concept (usually lecture), illustrate the new concept with problems, assign homework, then do some homework problems. While many math classrooms today are more progressive and reflect changes in mode of instructional delivery (hands-on activities, collaborative groups, investigations), lessons still tend to "aim for the average student" where instruction culminates with an evaluative activity and the attitude that "it's just too bad ten out of twenty-five students didn't pass the test" prevails. The teacher still goes on to the next lesson or unit. Standards-driven teaching provides opportunities for *all* students to meet standards and "pass the test." Teachers need to have a strategy to help each and every student to succeed.

Teachers need to ensure that the classroom environment is safe, nurturing, and facilitates active learning and supports investigations and rich experiences that are contextually grounded for student learning. Accommodations must be made, and differentiated teaching needs to occur to ensure that all students meet standards. A major challenge for teachers and teaching is determining the "when" and "how" to provide additional experiences for those students who need "supplemental help" to learn.

Teaching needs to be student-centered and accept and respect the cultural, intellectual, and diversity of learning styles in the classroom. Teaching needs to take students from where they are and build upon their prior knowledge, interests, and levels of understanding. As such, teachers need effective strategies based on cognitive and metacognitive practices that can help all students come to view mathematics not as an isolated set of rules to be memorized, but as a composite of ideas, mathematical domains, and procedures that are to be used as tools for understanding their world and creating new knowledge relevant to current and future needs.

STANDARDS-BASED CURRICULUM

A standards-based curriculum includes the learning experiences and sequence of units that help students achieve standards. The HCPS III mathematics benchmarks and the General Learner Outcomes are the foundation upon which the curriculum is built.

As part of a statewide educational system, parents and students deserve assurances that the content expectations at one school are the same as any other school; furthermore, if a student transfers from one school in Hawaii to another, there should be minimal disruption in his/her learning. To this end, the HCPS III Mathematics Benchmark Maps provide a clustering of the grade-level/course benchmarks and a quarterly sequencing of those clusters. (See

Appendix B.) At least on a quarterly basis, there is consistency across the state in the benchmarks that are being taught and assessed.

Support tools and assessments developed by the Department will be based on the benchmark maps. For instance, the Student Assessment Section has developed formative quarterly assessments such that the quarter one assessment provides formative data on students' proficiency of the benchmark maps' quarter one benchmarks.

While the benchmark maps only include a clustering and sequencing of the benchmarks, they do not include other elements of a curriculum such as related assessments and instructional experiences. To develop a complete curriculum aligned to the benchmark maps, additional planning by the classroom teacher is needed. Ideally, curriculum planning should be done through a collaborative effort.

At the elementary level, articulation across grade levels is essential to ensure that curricula implemented at each grade supports and enhances student achievement and that safety nets are built to ensure that all children meet or exceed standards. At the secondary level, mathematics departments should engage in articulation to assure consistency in curriculum among teachers teaching the same course. Within the complex of schools, there needs to be articulation pre-kindergarten through grade 12 and beyond to ensure that the expectations of each receiving school is communicated with each sending school and that students' needs are met at each grade level or services identified so that "no child is left behind." It is through this complex-wide articulation that each level can become aware of what curricular tools are being used and conversations initiated and maintained on professional improvement topics such as "which tools seem to best help improve student achievement towards meeting standards."

STANDARDS-BASED ASSESSMENT

In a standards-based system, assessments are critical because they represent the targets for instruction and they focus attention on what is needed for all students to meet standards. Assessment should not be confused with evaluation. Anne Davies (2000, 1) makes the distinction clear: "When we assess, we are gathering information about student learning that informs our teaching and helps students learn more. We may teach differently, based on what we find as we assess. When we evaluate, we decide whether or not students have learned what they needed to learn and how well they have learned it. Evaluation is a process of reviewing the evidence and determining its value."

In standards-based education process, assessment is critical to learning because it provides necessary feedback during the learning so students know how to get better or improve their work. It is important for students to know ahead of time what criteria will be used to judge their work and what the descriptors of quality look like so that they can share the responsibility for their own learning. The process of setting and using criteria for judging work should involve students and their teacher. Collaborating and agreeing on what the "standard" should contain makes clear to all involved what the work must include and look

like. When students are involved in this process, there tends to be commitment to producing quality work. The logical next step to this process is to share the "judging" or evaluative role with students and encourage student self-assessment.

In standards-based education, assessment is critical to teaching because it provides important feedback to teachers so that they can think about what needs to change in instruction to help students learn. Hawaii's standards-based assessment system consists of two parts—classroom assessment and the statewide assessment. Classroom assessments should provide information on where students are with respect to the standard or goal set for the lesson or unit. If not all students produced the quality of work the teacher was looking for, what had to change in the lesson? Reflection on the results of teaching is an essential piece for improving the teaching-learning process. Professional development to help teachers refine their reflection should emphasize that

1. Teachers can help each other "think about" what went on in instruction—analysis of the instructional strategy used and resultant student learning should be the basis for teachers' collaboration on their craft. Teachers working together to go over the instructional sequence, analyze student work, and share ideas of what the sequence should be to improve achievement helps them reflect on and improve their own teaching.

2. Assessment is an integral part of instruction. Assessment looks just like instruction; it can't be separated from instruction. Teachers have difficulty with this concept since they expect to see an assessment task that is different from an instructional task. When they can't see the difference, the first reaction is generally one of confusion. When they realize that the only thing that separates assessment from instruction is the purpose for which the task is being used, it's a major "aha."

3. If learning had not occurred as planned during the instructional setting, what might the interventions look like, when could the interventions occur (after school, during school, on Saturdays, etc.), and who could provide the interventions (tutor, classroom teacher, parent, volunteer, etc.)? If someone other than the teacher could be providing these interventions, what background must he/she have, what tools should he/she use, and what role does the teacher play?

PURPOSES FOR FORMATIVE CLASSROOM ASSESSMENT

Monitoring Students' Progress

Teachers monitor students' progress to understand and document their growth in relation to the standards and to provide students with relevant and useful feedback about their work and progress. If students are to successfully meet the HCPS III mathematics standards, assessment monitoring practices need to change. These changes require shifting

- Toward judging the progress of each student's attainment of the standards and away from assessing students' knowledge of specific facts and isolated skills.

- Toward communicating with students about their performance in a continuous, comprehensive manner and away from simply indicating whether or not answers are correct.
- Toward using multiple and complex assessment tools (such as performance tasks, projects, writing assignments, oral demonstrations, and portfolios) and away from sole reliance on answers to brief questions on quizzes and chapter tests.
- Toward students learning to assess their own progress and away from teachers and external agencies as the sole judges of progress.

Making Instructional Decisions

Teachers are able to make appropriate instructional decisions when they have a good understanding of what their students know and can do. Evidence of learning is used in three ways: (1) to examine the effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions; (2) to make instruction more responsive to students' needs; and (3) to ensure that every student is successful in meeting the appropriate standards. If students are to successfully meet the HCPS III mathematics standards, assessment practices to support instructional decisions need to change. These changes require moving

- Toward integrating assessment with instruction (to provide data for moment-by-moment instructional decisions) and away from depending on scheduled testing (generally useful only for delayed instructional decisions).
- Toward using evidence from a variety of assessment formats and contexts for determining the effectiveness of instruction and away from relying on only one source of information.
- Toward using evidence of every student's progress toward long-range goals in instructional planning and away from planning for content coverage with little regard for students' progress.

PURPOSES FOR SUMMATIVE CLASSROOM ASSESSMENT

Evaluating Students' Achievement

Evaluation is the process of determining worth or assigning a value to something, based on careful examination and judgment. If students are to successfully meet the HCPS III mathematics standards, how student achievement is evaluated must change:

- Toward comparing students' performance with performance criteria (e.g., rubrics) and away from comparing student with student.
- Toward assessing students' growth in relation to meeting the standards and away from assessing student knowledge of specific facts and isolated skills.
- Toward verifying student achievement based on balanced, multiple sources of information and away from relying on only a few, narrowly-conceived sources of evidence about student learning.

- Toward profiles of achievement based on publicly-stated criteria and away from single letter grades based on variable or nonpublic criteria.

Reporting Progress and Status

Elementary school students receive Progress Reports at the end of quarters one and three to inform parents of their child's progress toward meeting proficiency of the benchmarks for those quarters. Students also receive Status Reports at the end of the semester and the end of year to inform parents of their level of proficiency of the benchmarks for the respective semester. The "grades" on these reports are based on evidence showing what a student knows and is able to do in terms of the benchmarks. Multiple sources of evidence (e.g., observation checklists, student work samples) collected during the course of a year yield a more comprehensive, ongoing picture of student learning and academic progress towards achieving the standards.

While the standards-based report is not in effect for secondary school students, grades should still be based on achievement of the benchmarks. The emphasis on grading should be placed on a preponderance of evidence and multiple sources of evidence of the student meeting proficiency of the benchmarks, and not on a simple averaging of all the scores the students received throughout the quarter.

Evaluating Programs

A program evaluation uses student performance data with other evidence to judge the quality and success of the program. In addition to the evaluation of students' learning, program evaluations include information about other important elements, such as goals, curriculum materials, instructional methods, a student's opportunity to learn, and responsibilities of teachers and administrators.

STATEWIDE ASSESSMENT: THE HAWAII STATE ASSESSMENT

The Hawaii State Assessment (HSA) is administered annually to students in Grades 3, 4, 5, 6, 7, 8, and 10 and provides a one-time snapshot of what students know and are able to do. One part of the mathematics session of the HSA measures students' achievement as the Terra Nova test. This part of the HSA includes multiple-choice questions only. Students receive norm-referenced stanine and percentile rank scores that provide a comparison of their performance to that of thousands of other students in the same grade who took the Terra Nova session under the same testing conditions.

The other part of the HSA is standards-based and measures how well students understand mathematics concepts and skills though multiple-choice and constructed-response formats. Constructed-response questions require students to construct, produce, or create a written response. The questions directly measure all five of the HCPS III mathematics strands and their related content standards. In grade 10, questions measure proficiency of the standards from the Pre-Algebra and Algebra 1 courses. Each student receives a single proficiency level score that indicates his or her level of attainment for all five mathematics strands.

The following examples are sample items from the standards-based session of the HSA. More examples may be found on the HSA Released-Item website: http://www.hsaitems.org.

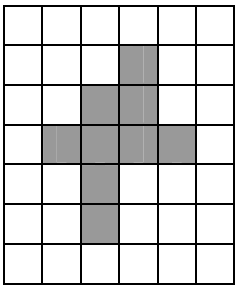
MULTIPLE-CHOICE

On the nature field trip, Chelsea and Lin saw 24 geckos and 10 butterflies. How many more geckos than butterflies did they see?

A 10
B 14
C 34
D 24

CONSTRUCTED-RESPONSE

Look at the shaded figure on the grid below. Complete the figure so that it has a line of symmetry. Draw the line of symmetry on the completed figure.



STANDARDS-BASED INSTRUCTION

Instruction in a standards-based system is designed to help all children achieve standards. The expectation that all students can learn challenging and rigorous content presents a challenge to teachers who must vary instruction in different ways and over different periods of time. To do so requires assessments to obtain information about where students are at, instruction adjusted to and informed by that information, the measurement of student progress by collecting evidence of learning in relation to the standards, and judging then reporting that progress to students, parents and other instructional personnel.

Planning for instruction requires a fundamental belief that *all* students can learn, and requires a strong mathematical background to be able to plan and adjust instruction to meet the varying needs of students. The following practices characterize what is meant by standards-based instruction.

1. Teachers believe that what they've planned will result in *all* students learning and achieving challenging standards. This belief and attitude that all students can learn and meet the challenging content of the standards outweighs the best planning for instruction that may have taken place. If the teacher doesn't hold this fundamental belief, that *all* can learn, then, like the self-fulfilling prophecy, not all children will

learn. A related challenge for teachers is to foster the belief in students and their parents that all students *can* learn the challenging content of the standards.

2. Teachers make accommodations for differences in styles of learning, relevant prior knowledge, and learning pace. Children are all different—they come with different experiences and knowledge, they have different needs, they learn at different rates, and they come with different interests. In a standards-driven system, teachers need to differentiate instruction in order to accommodate the needs of all students.

3. Teachers emphasize and incorporate the General Learner Outcomes and related mathematics process standards throughout the instructional process. Instruction needs to focus on conceptual understanding and the development of critical thinking processes that research has shown support the development of mathematical proficiency and further enable students to apply what they've learned to new situations and problems. Helping students to become self-directed learners who produce quality work and can effectively communicate ideas and use technology for multiple purposes prepares them for a highly technical future in which creative thinking, quick access to resources, and reliance on self are among just a few aspects of living in a world we can't even picture today.

4. Teacher create a learning environment that supports and enables student collaboration; "doing" mathematics; and free exchange of ideas exemplified by conversation, debate, logical arguments, justifications, predictions, etc.

5. Teachers use instructional strategies that are research-based, thoughtfully chosen with students and outcomes in mind, and effectively carried out with the additional purpose of assessing results of strategies to continually improve instructional delivery and student learning.

6. Teachers use instructional tasks that are rich, relevant, aim for "deep" learning and result in student work that contains evidence of the quality described for meeting and exceeding expectations. Rich tasks should address multiple standards and may provide the context for integrating multiple content areas and disciplines. Tasks should be crafted and sequenced to form rich instructional units that are purposeful and result in culminating activities where students can showcase their learning with meaningful audiences.

THE RELATIONSHIP BETWEEN THE MATHEMATICS STANDARDS AND THE GENERAL LEARNER OUTCOMES

Through the teaching and learning process, the General Learner Outcomes must be addressed in all mathematics classrooms because these outcomes are the center of standards-based learning. The mathematics content, process, and performance standards support the learner's progress towards achieving these six Outcomes:

1. **Self-Directed Learner** (The ability to be responsible for one's own learning.)
   Students must be encouraged to question the content (making meaning for themselves), question the teacher (clarification), question their peers (requesting clarification or justification for the ideas of their peers), and themselves (self-reflection, self-assessment). This empowers students to be responsible for their own learning.

2. **Community Contributor** (The understanding that it is essential for human beings to work together.)
   In order to be a successful contributor to society, students must have the opportunity to learn cooperative and collaborative skills and participate in cooperative and/or collaborative groups. Presentations and other classroom discourse should serve as the means for active practice in respecting one's rights as a human being as well as the rights of others.

3. **Complex Thinker** (The ability to be involved in complex thinking and problem solving.)
   Students must be provided opportunities to engage in developmentally-appropriate and challenging tasks. Students need to be allowed to be creative in their problem solving, but at the same time be able to justify their thinking and solution. Just as the level of mathematics needed for intelligent citizenship has increased, so too has the level of mathematical thinking and problem solving needed to be successful in the real world.

4. **Quality Producer** (The ability to recognize and produce quality performance and quality products.)
   Students must be provided opportunities to be active participants in setting criteria for a variety of tasks. Active participation in setting the criteria encourages student understanding (recognizing what it will take to meet the standard), ownership (living up to the criteria they set for themselves), and successful achievement of meeting the standard(s).

5. **Effective Communicator** (The ability to communicate effectively.)
   Communication is an essential part of mathematics. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing. Listening to

others' explanations gives students opportunities to develop their own understandings.

6. **Effective and Ethical User of Technology** (The ability to use a variety of technologies effectively and ethically.)
   Technology is essential in teaching and learning mathematics. Technology can enhance the learning of mathematical concepts. The graphic power of calculators and computers allows students access to powerful visual models as well as enriches the range and quality of investigations.

While the General Learner Outcomes are meant to be general outcomes for all students in all content areas, they align closely with the mathematics Process Standards. The table below shows how achieving the GLOs will also result in achieving the Process Standards.

General Learner Outcomes	Mathematics Process Standards
1. Self-Directed Learner • Sets priorities and establishes achievable goals and personal plans for learning • Plans and manages time and resources to achieve goals • Monitors progress and evaluates learning experiences	**Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics **Problem Solving** • Monitor and reflect on the process of mathematical problem solving
2. Community Contributor • Respects people's feelings, ideas, abilities and cultural diversity • Cooperates with and helps and encourages others in group situations • Understands and follows rules of conduct • Analyzes conflict and applies methods of cooperative resolution • Demonstrates responsible and ethical behavior in decision making • Responsibly implements a solution	**Communication** • Organize and consolidate their mathematical thinking through communication • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
3. Complex Thinker • Applies prior learning experiences to new situations • Considers multiple perspectives in analyzing and solving a variety of problems • Generates new and creative ideas and approaches to developing solutions • Evaluates the effectiveness and ethical considerations to a solution and make adjustments as needed	**Communication** • Organize and consolidate their mathematical thinking through communication **Representation** • Create and use representations to organize, record, and communicate mathematical ideas • Select, apply, and translate among mathematical representations to solve problems **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics • Make and investigate mathematical conjectures • Develop and evaluate mathematical arguments and proofs • Select and use various types of reasoning and methods of proof

General Learner Outcomes	Mathematics Process Standards
Complex Thinker, con.	**Making Connections** • Recognize and use connections among mathematical ideas • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole • Recognize and apply mathematics in contexts outside of mathematics **Problem Solving** • Build new mathematical knowledge through problem solving • Solve problems that arise in mathematics and in other contexts • Apply and adapt a variety of appropriate strategies to solve problems

INSTRUCTIONAL MATERIALS FOR MATHEMATICS

In order for any school mathematics program to achieve the goals of the standards, both teachers and students will require access to instructional materials that are accurate in mathematics content, clear in their presentation of mathematical understandings and processes, age-appropriate for the children who will use them, suitable for the local community, and consistent with the HCPS III mathematics standards.

Selecting the "right" textbook is not a simple task because publishers do not create textbooks that are designed specifically for Hawaii's standards. As part of the process to select mathematics materials, the materials must be judged on how well they help students to achieve proficiency of the HCPS III mathematics standards. In addition, any benchmarks that are not address in the instructional materials must be identified, and supplemental lessons for those tasks must be identified or created.

The selection of instructional materials should be based on set criteria. See Appendix E for templates of suggested criteria to review instructional materials. The Office of Curriculum, Instruction and Student Support conducted an official review of mathematics instructional materials in school year 2007-08.

There is no one set of instructional materials that will be sufficient to meet classroom instructional needs and the standards. It is essential that teachers understand the mathematics standards and become critical consumers. Instructional materials alone cannot ensure that learning will take place.

INTEGRATION: LITERACY AND MATHEMATICS

Communication, one of the process standards, is a vital part of mathematics and mathematics education. Instructional programs should allow all students to: 1) order and unify their mathematical thinking through communication; 2) articulate their mathematical thinking comprehensibly and with clarity to peers, teachers and others; 3) analyze and assess the mathematical thinking and strategies of others; and 4) use the language of mathematics to convey their mathematical ideas precisely.

Reading and writing in mathematics are an integral part of standards-based mathematics curricula. Reading and writing activities can help students analyze, clarify, interpret, and articulate their mathematical ideas, making mathematics more meaningful and motivating. Research suggests that the most logical place for instruction on reading and writing strategies is in the content areas rather than in separate reading classes.

Writing provides opportunities for students to elucidate their own understanding of mathematics and perfect their communication skills. Competency in writing can only be realized through active practice and solving mathematics problems is a natural way to achieve this.

Students who are provided with opportunities, continued support, and a lot of encouragement for speaking, writing, reading, and listening in mathematics classes acquire two benefits, they communicate to learn mathematics, and they learn to communicate mathematically.

4. BIBLIOGRAPHY, RESOURCES, AND GLOSSARY

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RESOURCES

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National Council of Teachers of Mathematics. Reston, VA: NCTM, 2003.
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National Council of Teachers of Mathematics. Reston, VA: NCTM, 2001.
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Aunty Math is an interactive web site that encourages K-5 students to submit answers and solution strategies that help solve mathematics challenges.

Balanced Assessment
http://balancedassessment.concord.org/
A library of over 300 mathematics assessment tasks developed during the project remains freely available through this web site. Teachers may use these materials in their own classrooms at no cost.

Census in Schools
http://www.census.gov/dmd/www.teachers.html
Offers teaching tools, training opportunities, and resources that help teachers promote data literacy and increase awareness of the Census Bureau and its products and programs.

Cyberchase
http://www.pbskids.org/cyberchase/classroom
Cyberchase is a new Public Broadcasting System (PBS) animated, adventure TV series about problem solving and math. On the web, students play interactive games and teachers can access lesson plans.

edHelper
http://edHelper.com
Dynamic Worksheets contains standardized test prep materials, mathematics puzzles, critical thinking problems, word problems, and dynamic worksheets.

Eisenhower National Clearinghouse
http://www.goenc.org
This website contains many resources for K-12 mathematics and science teachers.

Massachusetts Comprehensive Assessment System (MCAS)
http://www.doe.mass.edu/mcas
This website contains released test items from Massachusetts' state assessment, including their portfolio-based alternate assessment.

Mathematics Models I by Paul Griffith
http://www.europa.com/~paulg/mathmodels/math_models.shtml#VMI-1
Parents, teachers and students have access to animated models for a variety of mathematics concepts such as the four-basic operations, properties, building intuitions about fraction operations, integers, and more.

Math Puzzles
http://www.mathpuzzle.com
This website contains a collection of interactive games that give practice in basic logic and arithmetic.

National Council of Teachers of Mathematics
http://www.nctm.org
The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring mathematics learning of the highest quality for all students.

National Assessment of Educational Progress (NAEP) –Mathematics
http://nces.ed.gov/nationsreportcard/mathematics/
This website provides released mathematics test items available for classroom use. Student work samples with assessment rubric and evaluator commentary for each level of proficiency are provided.

National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html
This website consists of virtual manipulatives such as base-10 blocks, pentominoes, integer chips, histograms, variety of graphs and more to solve a variety of problems that span the five HCPS III strands. NCTM standards-linked lessons for parents, teachers and students using the virtual manipulatives may also be accessed. Manipulatives and lessons are organized by grade level clusters: K-2, 3-5, 6-8, and 9-12.

Performance Assessment Links in Mathematics (PALM)
http://palm.sri.com
This website is an on-line, standards-based, resource bank of mathematics performance assessment tasks indexed with the National Council of Teachers of Mathematics (NCTM) standards.

Problem Corner
http://Math.smsu.edu/~les/POTW.html
This website offers mathematics problems for high school and college-level students. It is on the Southwest Missouri State University's Department of Mathematics web site.

The U.S. Mint 50 Quarters Program
http://www.usmint.gov/mint_programs (click on 50 Quarters Program, then click on Education Initiative)
The U.S. Mint offers free, downloadable lesson plans on its 50 State Quarters Program, for grades K-6. The lessons teach the history of each state, as well as mathematics lessons using the new coinage.

GLOSSARY

The glossary contains terminology that may be found in the Hawaii Content and Performance Standards III Mathematics Standards document as well as within the Mathematics Curriculum Framework.

algorithm—A precise, step-by-step method or set of rules for solving problems.

array—The arrangement of objects in rows and columns (e.g., two rows and three columns of stickers equals six stickers).

associative property of addition (APA)—The way in which three numbers to be added are *grouped* two at a time and does not affect the sum (e.g., (3 + 5) + 2 = 3 + (5 + 2) = 10).

associative property of multiplication (APM)—The way in which 3 numbers to be multiplied are *grouped* two at a time and does not affect the product (e.g., (3 x 5) x 2 = 3 x (5 x 2) = 30).

assessment—The process of gathering evidence about a student's knowledge of, ability to use, and disposition toward mathematics and of making inferences from the evidence for a variety of purposes.

attribute—A quality that is thought of as belonging to a person or thing (i.e., characteristics such as size, shape, color, thickness).

back-to-back stem-and-leaf plot—A plot drawn with two stem-and-leaf plots that share the same stem in order to compare two sets of data; for example, the following data sets are being compared:

Data set 1: 7, 10, 11, 14, 16, 16, 18, 20, 21, 21, 21, 23, 25, 28, 28, 29, 30, 32, 32, 33, 41
Data set 2: 12, 15, 16, 18, 20, 23, 25, 26, 26, 27, 27, 27, 29, 29, 30, 30, 31, 31, 34, 37, 39, 40, 48

7	0	
8 6 6 4 1 0	1	2 5 6 8
9 8 8 5 3 1 1 1 0	2	0 3 5 6 6 7 7 7 9 9
3 2 2 0	3	0 0 1 1 4 7 9
1	4	0 8

benchmark—A specific statement of what a student should know or be able to do at a specific grade level or course

benchmark maps—a quarterly sequence of clustered benchmarks to be covered within a grade level or course that provide consistency in identifying when benchmarks will be addressed as schools develop curriculum maps

bivariate data—Data in two variables

cardinal direction (also **cardinal point)**—compass points of north, south, east, and west

Cartesian product—The product set that contains all possible combinations of one element from each set (e.g., two T-shirts (T), one red and one blue; three pairs of pants (P), one navy, one white, and one black; results in 6 combinations – red T, navy P; red T, white P; red T, black P; blue T, navy P; blue T, white P; blue T, black P)

certainty—In the study of probability, the belief that an event will always occur (probability of 1) or never occur (probability of 0)

coherence—The quality of logical connection and orderly relationship of parts

combination—An arrangement of *r* objects from a group of *n* objects where the order is not important

commutative property of addition (CPA)—Changing the *order* of two numbers (addends) to be added does not change the sum (e.g., 4 + 7 = 7 + 4 = 11)

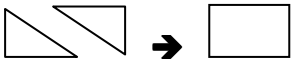
commutative property of multiplication (CPM)—Changing the *order* of two numbers (factors) to be multiplied does not change the product (e.g., 4 x 7 = 7 x 4 = 28)

complementary events—Two events such that one of the events is based on the other event not occurring (e.g., the events "picking an odd number" and "picking an even number" are complementary when picking a whole number because all whole numbers are either odd or even)

complex number—Any number that can be written in the form a + b*i*, where *a* and *b* are real numbers and ***i*** is the imaginary unit.

composing numbers—Adding two or more numbers; bringing together the objects used to represent two or more numbers.

composing shapes—Combining two or shapes to form a single shape; for example:



composition of transformations—A combination of transformations performed one after the other

composite number—A number which has factors other than itself and one (e.g., 6 is a composite number because it has factors, 2 and 3, in addition to 1 and itself)

congruent—when two or more figures have the exact same shape and size; symbolically, ≅

content standard—A broad statement of what a student needs to know or be able to do at the end of K-12 schooling

concepts—General and fundamental ideas–for example, the ideas that are needed to guide reasoning, problem formulation, and problem solving in non-routine situations.

conjecture—An unproven statement that is based on observations

cooperative learning or **collaborative learning**—Students of varying abilities and interests working together in small groups to solve a problem, complete a project, or achieve a common goal

counting on—A strategy for adding two numbers in which the counter uses one of the numbers as a starting point and continues counting by the amount stated by the second number (e.g., to find the sum of 6 + 5, the counter will start from "6" then add 5 by counting, "7, 8, 9, 10, 11")

criteria—Clear and specific statements that specify the dimensions or characteristics for judging student work

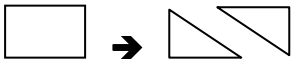
Cross-section—The intersection of a three-dimensional figure and a plane; the plane figure obtained by cutting a plane through a three-dimensional object (e.g., the cross-section obtained by cutting a plane parallel to the base of a cone is a circle)

curriculum (plural:**curricula)**—A plan of instruction that details what students are to know, how they are to learn it, what the teacher's role is, and the context in which learning and teaching will take place

customary units—United States standard units of measure (e.g., inch, foot, ounce, pound)

decomposing numbers—Representing a number with two or more of its addends (e.g., the number 35 can be decomposed into 20 + 10 + 5); representing a number with objects, then separating the objects into two or more groups; breaking apart a number in different ways (e.g., 23 ones = 1 ten and 13 ones OR 2 tens and 3 ones).

decomposing shapes—Subdividing a shape into two or more shapes



dependent events—two or more events in which the occurrence of one event effects the outcome of the next event (e.g., if A and B are dependent events and A occurs first, the P(A and B) = P(A) x P(B, once A has occurred)

describe a pattern—Explain how to replicate or continue the pattern

dilation—A transformation where a figure is proportionally magnified (enlarged) or reduced (shrunk) by a certain scale (e.g., a 50% dilation will reduce a figure; a 200% dilation will magnify a figure); a proportional shrinking or enlargement of a figure

discriminant—The value, $b^2 - 4ac$, of the quadratic equation $ax^2 + bx + c = 0$

disjoint sets—Two sets whose intersection is the empty set (e.g., {even numbers} and {odd numbers} are disjoint because they do not have any elements in common)

distributive property of multiplication over addition (DPMA)—Adding up the terms first then multiplying by the factor results in the same answer as multiplying each term by the factor, then adding the results (e.g., $2\ (3 + 5) = 2(8) = 16$; $2\ (3 + 5) = 2 \times 3 + 2 \times 5 = 6 + 10 = 16$)

extend a pattern—Continue a pattern by applying its rule to determine what comes next

evaluation—The process of determining the worth of, or assigning a value to something on the basis of careful examination and judgment

expected value—The amount that is expected to be gained using the calculation for average expected payoff

fact family—A set of four related equations involving addition and subtraction (or multiplication and division) that use the same three numbers (e.g., $3 + 4 = 7$, $4 + 3 = 7$, $7 - 4 = 3$, and $7 - 3 = 4$ make up a fact family; $5 \times 2 = 10$, $2 \times 5 = 10$, $10 \div 2 = 5$, $10 \div 5 = 2$ make up a fact family)

fairness—In the study of probability, when each event has an equally likely chance of occurring; when the expected value of each event are equal

Fibonnaci sequence—The unending sequence of integers (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, …) formed according to the rule that each integer is the sum of the preceding two

flexible—Able to switch from one form of representation to another

fluency—The ability to efficiently and effectively apply mathematics

flip—*see* ***reflection***

Fundamental Counting Principle—If an action can be performed in a ways and then, for each of these ways, another action can be performed in b ways, the two actions can be performed in $a \times b$ ways

function—Concept of a correspondence or relationship between two sets. A function f is a rule that assigns to each element x in set A exactly one element called $f(x)$ in set B

generalize—To state a rule that can be used to determine any term (such as the 100^{th}, 1000^{th}, or n^{th} term) in a given sequence

graphing technology—Technology, such as graphing calculators or computers, that allows users to create, explore, and analyze graphs and relate them to tables and equations

greatest integer function—A piece-wise defined function defined as the greatest integer less than or equal to x (e.g., f(1.4) = 1)

inclusion—The relationship between two sets such that one of the sets is a subset of the other set (e.g., if A = {whole numbers} and B = {even numbers}, then A is inclusive of B}

identity property—The idea that adding zero to any number does not change the number, and multiplying any number by one does not change the number

independent events—Two or more events are said to be independent if the outcome of one does not affect the outcome of the others; if A and B are independent events, then P(A and B) = P(A) × P(B)

inferences—Conclusions or assertions derived from evidence; deductions

inverse operations—two operations that have an opposite effect (e.g., adding and subtracting, multiplying and dividing, squaring and taking the square root)

irregular shapes—Shapes that do not have common names due to their unique designs

irrational number—A number which cannot be expressed in the form m/n, where m and n are integers and $n \neq 0$, e.g., $\sqrt{2}$, π

isometric drawing—A drawing using isometric dot paper to show perspective.

limit (limit concepts)—The fundamental concepts of calculus expressing the notion of tending to, or approaching, an ultimate value ($\lim_{n\to\infty} a_n = A$).

line of symmetry—A line that divides a figure into two congruent halves which are mirror images of each other; a line that divides a plane in half such that each set of corresponding points on opposite sides of the line of symmetry are equidistant from the line of symmetry

line symmetry—a property of a plane figure in which two halves of the figure that are separated by the ***line of symmetry*** are mirror images of each other

manipulative—Any physical object that can be used to represent or model a problem situation or develop a mathematical concept, e.g., blocks, toothpicks, coins, etc.

matrix—A rectangular array of numbers in rows and columns

mental math—Calculations that can be done without writing it down or using any tools

measures of central tendency—Mean (the average of a set of data), median (the middle or average of the two middle items of a set of data which has been arranged in order), and mode (the most frequently appearing items in a set of data)

metacognition—Thinking about one's own thinking

nets—Surfaces of a three-dimensional solid drawn in two dimensions, with indicated edge lines where folding could occur

norm-referenced test—A test that compares quantitative scores (such as the number of correct responses) to a normal distribution of such scores for the same age or grade

open-ended questions—Tasks that allow for various acceptable answers and/or allow for multiple approaches to an effective solution; tasks that engage students in interesting situations and allow students at many levels of understanding to begin working on the problems, make their own assumptions, develop creative responses, and effectively communicate their solutions

open sentence—An equation or inequality with at least one unknown quantity

parallel box-and-whisker plots—Two or more box-and-whisker plots drawn one above the other and sharing the same scale in order to compare sets of data

partitioning—Separating into two parts without counting

Pascal's triangle—A triangular array of numbers where the numbers in the array are the coefficients of the terms in the expansion of $(x + y)^n$ for n = 0, 1, 2, …

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \\ 1 & 4 & & 6 & & 4 & 1 \end{array} \text{ etc.}$$

pedagogical knowledge—The methods and strategies of teaching.

permutation—An arrangement of *r* objects from a group of *n* objects where the order is important

piece-wise defined—Functions which are defined by more than one equation, e.g.,

$$F(x) = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

plane figure—A two-dimensional figure

plane of symmetry—A plane that divides a three-dimensional figure into two congruent halves which are mirror images of each other

portfolio assessment—An assessment process based on the collection of student work (e.g., writing assignments, drafts, artwork, etc.) that contain skill competencies, exemplary work, etc. that show the student's developmental progress

prime number—A number whose only factors are one and itself (e.g., 17 is a prime number because its only factors are 1 and 17)

prior knowledge—The knowledge or experiences that a student has retained and brings to a new situation

problem solving—Engaging in a task for which the solution is not immediately known or not easily derived by applying a formula or ***algorithm***

quartile—One of four parts of data formed when a set of data is arranged in order and separated into four parts having an equal number of data

random sample—A sample chosen in a way that gives every member of the population an equally likely chance of being selected

range—The difference of the largest and smallest values in a set of data

rate of change—The quotient of the vertical (output) change divided by the horizontal (input) change

rational number—A number which can be expressed in the form m/n, where m and n are integers and $n \neq 0$

real numbers—A number which is ***rational*** or ***irrational***

recursive—The nature of a pattern in which each succeeding terms is formulated from one or more previous terms (e.g., Each number in the Fibonacci sequence, 1, 1, 2, 3, 5, 8, 13, …, is the sum of the two previous terms)

reflection (**flip**)—A transformation creating a mirror image of a figure on the opposite side of a given line, the ***line of symmetry***

reflectional symmetry—When all points of a figure on one side of the ***line of symmetry*** are equidistant from their corresponding points of the figure's image on the other side of the line of symmetry

representing data—Grouping or displaying collected data in some organized way (e.g., table, bar graph, histogram, line plot, circle/pie graph, stem-and-leaf plot, box-and-whiskers plot, scatter plot, line graph)

rotation (**Turn**)—Transformation in which a figure is turned a given angle and direction around a given point

rotational symmetry—When all points of a figure coincide when the figure is rotated *x* degrees about an axis of rotation

rubric—Tool to assess the quality of students' achievement of the ***benchmarks***

Sample Performance Assessment—A generalized description of how students can demonstrate significant aspects of the ***benchmark***

similar—When two shapes have corresponding angles that are equiangular and the ratio of corresponding side lengths are the same

sinusoidal data—Data that repeats itself in regular intervals (e.g., the tides; sine function)

slide—see ***Translation***

spherical coordinates—Locations in three-dimensional space (e.g., (x, y, z))

spread—The distribution of a set of data; a statement of the minimum value and the maximum value of a set of data

square number—Any of the numbers 1, 4, 9, 16, … such that the nth term is n^2; number associated with the number of dots in a square array

standard—A statement about what is valued, what students should know and be able to do.

standard algorithm—A precise, step-by-step set of rules or procedures that are followed to perform calculations

standard deviation—A measure of dispersion in a frequency distribution; the square root of the mean of the squares of the mean differences

step function—*See* ***Piece-wise defined function****, also* ***Greatest integer function***

strand—Organizes the standards into big ideas. In mathematics, the five **strands** are: (1) Number and Operations, (2) Measurement, (3) Geometry and Spatial Sense, (4) Patterns, Functions, and Algebra, and (5) Data Analysis, Statistics, and Probability

task—An authoritatively-specified or assigned, purposeful, contextualized activity

topic—Organizes the ***benchmarks*** into related ideas

transformation—The operation that moves a pre-image onto an image; s*ee also* ***Reflection, Rotation, Translation***

translate among tables, graphs, and equations—To switch back-and-forth between different representations; to use one representation to make another

translation (**slide)**—A transformation that moves every point on a figure a given distance in a given direction

triangular number—Any of the numbers, 1, 3, 6, 10, ..., where the n^{th} term is the sum of the arithmetic series $1 + 2 + 3 + \ldots + n$; a number associated with the number of dots in a triangular array

truncation—The remaining three-dimensional object when part of the three-dimensional object is removed

turn—see ***Rotation***

vary directly—When one variable (quantity, value) increases, the other increases proportionally

vary inversely—When one variable (quantity, value) increases, the other decreases proportionally

vector—A quantity that has both direction and magnitude, and is represented by an arrow drawn between two points

5. APPENDICES

Appendix A: Standards at-a-Glance

Appendix B: Benchmark Maps

Appendix C: Taxonomic Levels of the Mathematics Benchmarks

Appendix D: The Mathematics Strands and the General Learner Outcomes

Appendix E: Review Criteria for Mathematics Instructional Materials

APPENDIX A: STANDARDS-AT-A-GLANCE

STRAND	STANDARDS	• TOPICS
NUMBER AND OPERATIONS	**Standard 1: NUMBER SENSE** Understand numbers, ways of representing numbers, relationships among numbers, and number systems	• Numbers and Number Systems • Vectors
	Standard 2: OPERATION SENSE Understand the meaning of operations and how they relate to each other	• Operations • Operation Properties
	Standard 3: COMPUTATION STRATEGIES Use computational tools and strategies fluently and, when appropriate, use estimation	• Computational Fluency • Estimation • Vectors
MEASUREMENT	**Standard 4: FLUENCY WITH MEASUREMENT** Understand attributes, units, and systems of units in measurement; and develop and use techniques, tools, and formulas for measuring	• Measurement Attributes and Units • Measurement Tools and Techniques • Measurement Formulas
GEOMETRY AND SPATIAL SENSE	**Standard 5: PROPERTIES AND RELATIONSHIPS** Analyze properties of objects and relationships among the properties	• Geometric Shapes and Their Properties and Relationships
	Standard 6: TRANSFORMATIONS AND SYMMETRY Use transformations and symmetry to analyze mathematical situations	• Transformation • Symmetry
	Standard 7: VISUAL AND SPATIAL SENSE Use visualization and spatial reasoning to solve problems both within and outside of mathematics	• Visualization and Spatial Reasoning • Geometric Modeling
	Standard 8: REPRESENTATIONAL SYSTEMS Select and use different representational systems, including coordinate geometry	• Coordinate Geometry
PATTERNS, FUNCTIONS, AND ALGEBRA	**Standard 9: PATTERNS AND FUNCTIONAL RELATIONSHIPS** Understand various types of patterns and functional relationships	• Patterns • Function
	Standard 10: SYMBOLIC REPRESENTATION Use symbolic forms to represent, model, and analyze mathematical situations	• Numeric and Algebraic Representations • Rates of Change
DATA ANALYSIS, STATISTICS, AND PROBABILITY	**Standard 11: FLUENCY WITH DATA** Pose questions and collect, organize, and represent data to answer those questions	• Data Collection and Representations
	Standard 12: STATISTICS Interpret data using methods of exploratory data analysis	• Data Interpretation
	Standard 13: DATA ANALYSIS Develop and evaluate inferences, predictions, and arguments that are based on data	• Predictions and Inferences
	Standard 14: PROBABILITY Understand and apply basic notions of chance and probability	• Probability
The process standards of Communication, Connections, Problem Solving, Representation, and Reasoning and Proof are incorporated throughout the above mathematics standards.		

APPENDIX B: Mathematics Benchmark Maps

What is a benchmark map?
A benchmark map shows clusters of benchmarks that are sequenced by quarters. Benchmarks that are clustered together share Big Ideas or Major Understandings which are statements that provide the real world relevance of the cluster.

How can the benchmark map be used?
The benchmark map is used to plan and guide decisions about curriculum. The level of knowledge reflected in the benchmark guides decisions about the amount of time and practice needed for students to demonstrate proficiency on the benchmark. Schools that have developed consensus maps should check for alignment to the benchmark maps since future development of instructional and assessment tools will be aligned with the benchmark maps.

Where are the benchmark maps?
The benchmark maps can be accessed through the DOE website at http://doe.k12.hi.us. On the DOE homepage, drag the cursor onto the word "Standards" and continue dragging the cursor onto the word "HCPS III" on the pop-up menu. A sample of the grade 6 first quarter benchmark is provided below.

Grade: 6	Quarter: 1

Big Idea(s)/Major Understanding(s): *Students will understand that…*

- Data helps people to understand situations and make informed decisions.
- Graphical displays of numbers make it possible to spot patterns that are not otherwise obvious.

HCPS III Benchmarks:

- MA.6.11.1: Analyze how data collection methods and sample size can affect the results of data sets
- MA.6.12.1: Determine and interpret the measures of center (mean, median, mode) of a data set and explain what each measure indicates about the data set
- MA.6.12.2: Use a stem-and-leaf plot to analyze a set of data
- MA.6.13.1: Make inferences about a population based on the interpretation of a sample data set

Big Idea(s)/Major Understanding(s): *Students will understand that…*

- Understanding patterns can lead to educated predictions.

HCPS III Benchmarks:

- MA.6.9.1: Represent visual and numerical patterns with tables and graphs and generalize the "rule" using words and symbols
- MA.6.9.2: Describe simple one-step functions using words and symbols when given a table of "input" and "output" values

APPENDIX C: Taxonomic Levels of the Mathematics Benchmarks

Understanding the intended outcome of each benchmark is critical when designing assessment tasks that are supposed to measure students' proficiency of the benchmarks. Robert Marzano's taxonomic levels provides a way to establish levels of cognitive demand.

The following tools are provided in this appendix:

Marzano's Taxonomic Levels of Understanding (Cognitive Domain)

LEVEL 1: KNOWLEDGE RETRIEVAL	
Recognizing ▪ Select from a list	The student can identify or recognize features of information, but does not necessarily understand the structure of knowledge or is able to differentiate critical from non-critical components.
Recalling ▪ Give examples ▪ List/Name	The student can recall information, but does not necessarily understand the structure of knowledge or is able to differentiate critical from non-critical components.
Executing ▪ Read ▪ Perform mathematical operation (by following a set algorithm)	The student can perform a procedure without significant error but does not necessarily understand how and why the procedure works.

LEVEL 2: COMPREHENSION	
Integrating ▪ Describe and explain ▪ Explain the concept	The student can identify the basic structure of knowledge and the critical as opposed to non-critical characteristics of that structure.
Symbolizing ▪ Demonstrate and explain ▪ Diagram ▪ Illustrate/ describe how ___ is related to ___ ▪ Represent	The student can construct an accurate symbolic representation of knowledge, differentiating critical from non-critical elements.

LEVEL 3: ANALYSIS	
Matching ▪ Compare/Contrast ▪ Differentiate ▪ Find what is common among ___	The student can identify important similarities and differences in knowledge or skill.
Classifying ▪ Categorize	The student can identify superordinate and subordinate categories related to knowledge or skill.
Analyzing Error ▪ Determine reasonableness of information	The student can identify errors in presentation or use of knowledge.
Generalizing ▪ Create a rule ▪ Draw a conclusion	The student can construct new generalizations or principles based on knowledge.
Specifying ▪ Predict ▪ Determine what comes next/later	The student can identify specific applications or logical consequences of knowledge.

LEVEL 4: KNOWLEDGE UTILIZATION	
Decision Making ▪ Use ___ to determine ___ ▪ Judge the validity of ___	The student can use the knowledge to make decisions or the student is able to make decisions about the use of the knowledge.
Problem Solving ▪ Use___ to solve	The student can use the knowledge to solve problems or to solve problems about the knowledge.
Inquiring Experimentally ▪ Generate/Test hypotheses	The student can use the knowledge to generate and test hypotheses or to generate and test hypotheses about the knowledge.
Investigating ▪ Analyze using evidence ▪ Investigate	The student can use the knowledge to conduct investigations or to conduct investigations about the knowledge.

Marzano, R. J. (2001). *Designing a New Taxonomy of Educational Objectives*. Thousand Oaks, CA: Corwin Press.

Sample Page: Taxonomic Levels and Essence Statements for Selected Grade 6 Benchmarks

Code	Grade 6 Benchmarks	This means that the student can…
MA.6.1.1	Compare and order fractions, decimals, and percents *[Level 3: Analysis (Matching)]*	• Put a set of fractions, decimals, and percents in order from least to greatest or greatest to least.
MA.6.1.2	Explain and give examples of number theory concepts (e.g., prime factorization, common factors, greatest common factor, common multiples, least common multiple, divisibility) [*Level 1: Retrieval (Recalling)]*	• Explain (using drawings, words, numbers, and/or symbols) and give examples of number theory concepts: o Prime factorization o Common factors; greatest common factor o Common multiples; least common multiple o Divisibility
MA.6.2.1	Apply the order of operations when calculating with whole numbers *[Level 1: Retrieval (Executing)]*	• Follow the order of operations when calculating number sentences with more than one operation (with whole numbers only).
MA.6.2.2	Use the operation properties to simplify computations with fractions, decimals, and percents *[Level 4: Utilization (Decision Making)]*	• Use the operation properties (commutative, distributive, associative, identity, zero) to strategically simplify computations with fractions, decimals, and percents.
MA.6.3.1	Use estimation prior to computing with fractions and decimals and compare the estimation to the actual result *[Level 4: Utilization (Decision Making)]*	• Use an appropriate estimation strategy to mentally determine an answer, then perform the actual computation and compare the result to the estimation.
MA.6.3.2	Recognize situations in which it is more appropriate to estimate than to compute an exact answer [*Level 1: Retrieval (Recognizing)]*	• Indicates whether or not an estimate (rather than an exact answer) will suffice for a given situation.

The rest of the grade 6 benchmarks, as well as benchmarks for kindergarten to grade 8 and selected high school courses may be found on the mathematics website at http://isb.k12.hi.us.

APPENDIX D: The Mathematics Strands and the General Learner Outcomes

	GLO #1 Self-Directed Learner	GLO #2 Community Contributor	GLO #3 Complex Thinker	GLO #4 Quality Producer	GLO #5 Effective Communicator	GLO #6 Effective & Ethical User of Technology
Number and Operations	Sets priorities and goals Monitors progress • Reflects on what was learned • Updates priorities and goals as needed Plans and manages time and resources	Cooperates with and helps and encourages others in group situations Provides constructive feedback when evaluating the work of others Gives and accepts new ideas/strategies	Considers a variety of solution strategies when solving problems and performing calculations Checks for accuracy and reasonableness	Demonstrates proficiency in meeting the standards Understands the criteria for quality work and strives toward meeting (and exceeding) the criteria Improves and revises work when given the opportunity	Shows/explains the solution strategy Uses mathematical terms and symbols when communicating	Uses technology to: • Make sense of numbers • Understand operations • Perform complex calculations
Measurement			Chooses appropriate units and tools for measuring Questions the precision and accuracy of certain measurements Uses known measurements to determine other		Labels measurements with appropriate units Uses mathematical terms and symbols when communicating	Uses technology to: • Aid in solving measurement problems • Make measurements
Geometry & Spatial Sense			Poses conjectures about shapes Explores the properties of shapes; looks for relationships among shapes		Makes convincing and logical arguments Uses mathematical terms and symbols when communicating	Uses technology to: • Construct and manipulate shapes • Analyze the properties of shapes
Patterns, Functions and Algebra			Analyzes patterns: • Is there a pattern? • What comes next? • What comes later? • What's the rule?		Labels graphs, charts, and tables appropriately Provides appropriate representations and reasoning when communicating the solution to a problem	Uses technology to: • Develop an understanding of patterns • Explore the relationship between tables, graphs, and equations
Data Analysis, Stats & Probability			Analyzes data: • Selects appropriate representations • Hypothesizes • Poses conclusions • Questions results		Uses appropriate displays to represent and analyze data Labels graphs, charts, and tables appropriately Listens and observes in order to collect data	Uses technology to: • Display data in a variety of ways • Analyze data

APPENDIX E: Review Criteria for Mathematics Instructional Materials

IDENTIFICATION OF SUBMISSION	
Grade-Level(s) / Course	
Name of Publisher	
Title of Submission	
Copyright Date	
Price	

Instructions: Please choose the appropriate rating for each criterion. Total numerical score will be calculated on summary sheet. For any score that is 3 or above, explain fully in "documentation" section.

Explanation of Rating Code:

0=Not at all 1=Minimally 2=Adequately 3=Highly 4=Thoroughly

EVALUATION SCALE WORKSHEET		Points Possible	Points Received
Criteria A: Content	(Weight = 2)	64	
Criteria B: Instructional Strategies	(Weight = 2)	32	
Criteria C: Assessment	(Weight = 2)	16	
Criteria D: Specific Content Area	(Weight = 2)	72	
Criteria E: Non-Print Materials	(Weight = 1)	16	
	TOTAL POINTS	**200**	

CRITERION A: CONTENT

Explanation of Rating Code:

0=Not at all 1=Minimally 2=Adequately 3=Highly 4=Thoroughly

Extent to which:	Rating
Al. Alignment to Standards The program/text is aligned with the content standards and grade level/course benchmark maps.	
A2. Clarity of Objectives The instructional objectives are clearly stated and in consonance with the content standards and grade level/course benchmark maps and the sequence of activities build toward understanding of the mathematics concepts.	
A3. Variety of Strategies Different mathematical strategies are presented so that students may develop deep conceptual understanding .	
A4. Research Base The program or text is supported by scientifically-based research and by theoretical research.	
A5. Higher Order Thinking Problem-solving and decision making are incorporated into the program or text ranging from simple to complex. Reasoning is incorporated into the program or text, requiring students to think inductively and deductively and students are encouraged to seek information and solutions beyond what is in the text or program.	
A6. Relevance Student tasks and activities are worthwhile and relevant and content concepts are shown in relationship to real-life situations and other content areas.	
A7. Accuracy Content (e.g., concepts, maps, charts) is current and accurate.	
A8. Bias Content is free of bias and stereotype (e.g., gender, race, ethnicity) while illustrations such as drawings, photos, and artwork reflect sensitivity to diverse student populations.	

Documentation for Criterion A - Content:
A1:
A2:
A3:
A4:
A5:
A6:
A7:
A8:

CRITERION B: INSTRUCTIONAL STRATEGIES

Explanation of Rating Code:

0=Not at all 1=Minimally 2=Adequately 3=Highly 4=Thoroughly

Extent to which:	Rating
B1. Varied Teaching Strategies Suggested teaching strategies are appropriate and effective, and support differentiation for a range of learners.	
B2. Communication Oral and written communication are integrated into the program or text, requiring students to formulate, express and support generalizations, discuss relationships and/or cause and effect, and defend their ideas.	
B3. Critical Thinking Skills Activities suggested develop concepts and skills through active engagement; reading comprehension strategies and concept attainment strategies are provided; emphasis is on students actively involved in their learning, not just memorization of facts, dates, events and people; and students are encouraged to seek questions and responses beyond the text.	
B4. Learning Styles Students are encouraged to work collaboratively and independently.	

Documentation for Criterion B – Instructional Strategies
B1: B2: B3: B4:

CRITERION C: ASSESSMENT

Explanation of Rating Code:

0=Not at all 1=Minimally 2=Adequately 3=Highly 4=Thoroughly

Extent to which:	Rating
C1. Varied Assessments Assessment strategies are designed to connect to conceptual understanding; variety of questions, assessment activities for formative and summative assessment are included throughout; and variety of questions, assessment activities that address depth, breadth are included where appropriate.	
C2. Assessment Strategies Assessment methods match what is being assessed and major understandings and learning questions are mostly open-ended responses versus closed "right answer" responses.	

Documentation for Criterion C - Assessment:
C1: C2:

CRITERION D: MATHEMATICS

Explanation of Rating Code:

0=Not at all 1=Minimally 2=Adequately 3=Highly 4=Thoroughly

Extent to which:	Rating
D1. Engages students in learning mathematics via the five process standards: problem solving, communication, representation, reasoning and proof, and making connections.	
D2. Materials encourage student curiosity and support a collaborative classroom environment; provides a variety of contexts and first-hand experiences that engage students in learning.	
D3. Develops skills AND understanding by bridging abstract mathematical ideas to concrete/kinesthetic/visual representations.	
D4. Engages students with constructing mathematical understanding rather than only providing step-by-step algorithms.	
D5. Materials are coherent and make connections between key concepts and related concepts.	
D6. Materials present mathematical ideas without errors or misleading statements.	
D7. Applications of technology enhance learning.	
D8. Materials take into account student ideas and promote student reflection so that they can make sense of the mathematics concepts.	
D9. Materials provide teachers with the background knowledge to understand the mathematical content that is being taught.	

Documentation for Criterion D - Mathematics:
D1:
D2:
D3:
D4:
D5:
D6:
D7:
D8:
D9:

CRITERION E: NON-PRINT MATERIALS

Explanation of Rating Code:

0=Not at all 1=Minimally 2=Adequately 3=Highly 4=Thoroughly

Extent to which:	Rating
E1. Presentation and accessibility Content is well-organized and text is clear, concise, accurate, and grammatical while on-screen instructions are logical and/or intuitive and easy to use.	
E2. Accessibility Graphics and sounds help to facilitate comprehension and are relevant and aesthetically pleasing; unnecessary sounds and graphics are avoided; program runs smoothly without long delays; and help/tutorials are easily accessed, while program is in action or use.	
E3. Management Program allows customizing for individual learning needs and allows student to exit and resume at a later time. Program provides control of various aspects of the software (e.g., turning sound off) and enables student performance record to be kept and monitored by teacher, where needed.	
E4. Variety and Flexibility Program provides various types of printed reports for student and teacher (and/or parent) and has flexibility that allows for use in instruction/practice in different groupings (i.e. teacher facilitated with big group, student facilitated with small group, peer/pairs, individual student.)	

Documentation for Criterion E – Non-Print Materials
E1: E2: E3: E4: